MISTAKE IN GÖDEL

UNDECIDABILITY IN NATURAL LANGUAGE
FROM EPIMENIDES TO GÖDEL

HELIOS PAZOS

INTRODUCTION

We summarize below the origin and peripeteia of the objection formulated to Gödel related to his 1931's incompleteness theorem.

Attached is an English version provided to facilitate its access and diffusion, take notice that it might contain some language mistakes.

We guess that the fact that it's coming from a Civil Engineer and not from a Logic or Mathematician it's related to the lack of response obtained.

We stumbled into a simple problem, "The bounty", in "Humor and Games" magazine's 10th issue (May 1981), and applying basic logic rules we solved it in the

theorem previous explanations, evaluating as true a sentence that in all lights seemed to be but it publication's expected way. Afterwards, a global view of the problem showed that the solution found was unequivocally wrong.

It was surprising founding that the same easy and distracted mistake showed up on Kurt Gödel's incompleteness is not. Even when the mistake happens outside the theorem, its meaning it's changed when the sentence that affirms being not deductable it is not true.

It is not evident nor obvious which is the reasoning error, so when it was found it was communicated to the mentioned publication and made explicit on Uruguay's Engineering Magazine 1st issue from 1989.

In a happy occasion where we met with Jesus Mosterín in Spain, Gödel's work translator and compiler, he affirmed that the found mistake could not exist, and mentioned (with certain annoyance) that Gödel had been objected for several motives, but never because of the one signaled by this work. That was encouraging; as if the argument had been exposed it would have been refuted or confirmed already.

In 1992, this analysis was presented under "Undecidability in natural language - From Epimenides to Gödel", on the VIII Natural Languages and Formal Languages Congress, organized on Gerona's University by Barcelona's University, where it was accepted and published (Carlos Martín Vide, Editor). Prof. Francisco Rodríguez Consuegra (Logic Department – Philosophy Faculty – Barcelona University) was invited to this congress to speak at a seminar on the philosophical consequences of Gödel's theorem (MATHS AND PHILOSOPHY: THE KURT GÖDEL CASE)*. The case was shown to him remarking that it was unknown the error detected was transcendent or irrelevant, whereupon he said that if the analysis was correct, the consequences where catastrophic, and asked for two days to study it. At the end of this lapse he said that he did not found any incorrectness on the exposed, but he refused to believe that Gödel had been wrong, and offered an additional time to issue or rectify himself, thing that never happened.

Math's Institute from Engineering Faculty was consulted and twice the Logic Department from Humanities Faculty (both from the Republic's University) without getting any response. The same happened with several Logic magazines.

In 1995 the analysis was presented to Noam Chomsky, who said a priori that it could not be right, and recommended its presentation to the by then President of the Association for Symbolic Logic who, after studying it manifested: "It does not sounds convincing to me", without signaling any errors.

We naively believed Hans C. Andersen when he told us that it was enough that a child signaled that "the King is naked" for this to be shared by all. In this case, the questioned is not obvious, but we think that it deserves to be analyzed, and this is an invitation to walk this road, even when we don't know how far can lead us.

* Carlos A. Cardona, University of Bogotá: *"... it has been specially relevant the publication by Prof. Rodríguez Consuegra of some Gödel unedited documents."*

UNDECIDABILITY IN NATURAL LANGUAGE FROM EPIMENIDES TO GÖDEL

ABSTRACT

The logical analysis of a seemingly straightforward problem yields a simple and convincing solution to the liar's paradox and reveals an error in Gödel's presentation prior to his theorem on incompleteness.

Natural language provides us examples of sentences and/or pseudo-sentences that cannot be universally qualified as being true or false (E.g. Thursdays are yellow). If that character is not evident, as is the case, for instance, with Epimenides's statement, then accepting that it is either "True or False" not only leads to a theoretical conflict, but it also results in flagrant contradictions with reality.

Prior to his theorem on incompleteness, Gödel claims that a certain undecidable statement is true (which conveys a special meaningfulness to the fact that it is undecidable). The analysis shows that that statement is not true.

Keywords: Gödel, Godel, undecidability, recursiveness
2000 Mathematics Subject Classification: 03B65 - Logic of natural languages

Ing. Helios Pazos
Casilla postal 53114
Punta del Este-URUGUAY

UNDECIDABILITY IN NATURAL LANGUAGE
FROM EPIMENIDES TO GÖDEL

1 - LANGUAGE

Language is a marvellous tool that enables us to create, to think, to communicate (and probably to cause confusion) and to evolve and that has permanently surprised us since Plato's Cratilo. [7]

In general, there has been little or no awareness that natural languages resist (usually successfully) any attempt to box their contents.

In the 60's, researchers on Artificial Intelligence devoted to language problems stumbled against the semantic and pragmatic difficulties of open meanings; until then, the illusion had been that those issues could be easily solved.

2 - PROPOSITIONS THAT CAN BE QUALIFIED AS TRUE OR FALSE

We will see that the analyses of the conditions under which a proposition formulated in natural language can be qualified as being True or False are not obvious.

It is through the application of this analysis that we will object Gödel's statement in his explanation previous to his theorem on incompleteness, where he deducts that an undecidable statement is true.

Gödel's theorem has been objected for several reasons, but it has never been objected for stating that this statement is true. (Confirmed by Jesús Mosterín, compilator and translator of Godel's work.)

The quoted statement greatly resembles Epimenides's paradox. In that respect, J. Mosterín states:

> *"It is on the brink of the liar's paradox, but fails to fall in it (the statement ϕ* ***is*** *at the same time* ***true*** *and undeductible. This entails no paradox at all, but it does cause surprise)…"* - J. Mosterín [1; 48]

In none of the two cases can the statement be qualified as True or False; and although it may *a priori* seem true, the ϕ statement quoted is not so.

Not only have both Epimenides's and Gödel's propositions been treated as qualifiable as True or False, but the latter has even been accepted as **being** true.

3 – LOGICAL FRAMEWORK

The concept of the term True is in a delicate hinge between syntactics and semantics, from “true” in a formal system, where it can be derived from the basic premises (valid, analytical, demonstrable, [2;216]) to the term “true” used when a proposition in natural language is deemed consistent with a certain reality. (E.g. It is raining outside).

Qualifying something as true implies it agrees with certain rules of a specific system, within which the qualification is then valid. However, whenever we herein claim that a proposition (or pseudo-proposition) is Not Qualifiable as True or False, even when that conclusion is drawn through syntactic considerations, it implies attributing a lack of meaning that invalidates it for a similar qualification in any logical system and not in just a single specific one.

Hence, when we say Not Qualifiable as True or False, that implies not just adhesion to Aristotle’s binary logic, but it also intends to be valid in other systems. Pointing out that "Peanut butter is dodecaphonic" is not, in the Aymara trivalent logic, Qualifiable as True nor False nor Uncertain. Nor does it have any sense in Hans Reichenbach's "Logics of the probable", which assigns "truth" in a proposition a value between zero and one; attributing a certain value to truth or to the odds that the statement (or set of signs resembling a statement) has of being true; consequently, the Value of Truth in it is not Qualifiable either.

Briefly put, “Not Qualifiable as True or False” implies a disqualification and it attributes lack of sense both in its “natural interpretation” in the informal sphere of what we call “common sense” and in the formal domains of the logical systems we use.

4 – FORMAL FRAMEWORK

Although we know that the natural languages’ fluency and flexibility fail to make them more suitable for a rigorous analysis, those are the languages used to formulate the two cases discussed here (Epimenides and Gödel).

4.1 - Gödel’s challenged statement is expressed in the introduction and in the enunciation prior to developing his Incompleteness Theorem (attributing a special signification to it) and it is produced outside any formal system:

> *"Claiming that it is not deductible, the statement [R (q); q] immediately leads to saying that [R (q); q] is true, since [R (q); q] is not deductible (for it is not decidable). Hence, the undecidable statement in the PM system has finally been decided through meta-mathematical considerations.". The precise analysis of this strange situation leads to surprising results...".* Gödel [1; 59]

This positive statement, expressed in natural language, has never been challenged (J. Mosterín, see page 2).

4.2 - The other case, the liar’s antinomy, also formulated in natural language, has been exorcised in various ways:

- Gödel: *...but "the false positive statement in L" cannot be expressed in L, so its positive statement was in another language, and hence the paradox disappears...* [1;173]
- R. Carnap: [2;213]
- P. Watzlawic: ... *The paradox arises as a consequence of the positive statement's self-reflexiveness, i.e., confusing between member and class... Thus, Epimenides violates the core axiom of the theory of the logical types, that is, what comprises a whole collection (class) it cannot be a member of that said collection.* [4;91]

We will see that the problem may be solved in a simple and radical manner within the natural language and outside the formal systems.

Therefore, in the discussion below we will not be subject to any specific formalised ambit.

5 - CONDITIONS UNDER WHICH A PROPOSITION IS QUALIFIABLE AS TRUE OR FALSE

In his "Logical Syntaxis of Language", Carnap established the way to build the syntaxis of a formalised language, including the need to define the notion "well formed expression", i.e., enunciating the rules that form it.

Also in the field of formalised languages, Tarski described how to build a semantic and give a strict definition of the semantic notion of truth, logical consequences and validity.

Both were sceptical when considering the possibility of extending the application of the methods they had created to the natural languages.

It has been claimed that there are no significant theoretical differences between the formal and the natural languages – Montague [5]. However, together with the problem posed by D. Davidson,

> *All speaking subjects recognise a finite number of words as belonging to their lexicon, and they can apply a finite number of forming rules. Under these conditions, how is a subject capable of recognising that an endless number of phrases are "well formed", "grammatical" or "meaningful"? –* Davidson [6]

Carnap's positive statement continues to be valid for many reasons:

> *The grammatical syntax of a natural language is not capable of performing the task of eliminating all the cases of meaningless combinations of words.* Carnap [3; 13]

When expliciting the conditions required for a statement formulated in a natural language to be Qualifiable as True or False in a certain context, we must at least respect the basic requirements below:

A – Unequivocal identification of the subjects under discussion.
B – Unequivocal identification of what is attributed to these subjects.
C – Possibility of consistency among subjects and attributes.

This does not imply uniqueness in the analysis that determines or separates the terms of the proposition.

In: 2 + 3 = 5

- The subject may be 2 and the property may be that by adding 3, it makes 5.
- The subjects may be (2 + 3) and (5) and the property may be "having the same value".
- The subject may be 2 + 3 = 5 and the property may be "being a valid equality", etc.

If there is a way to describe or analyse the statement so it will meet the three conditions, then there is a Qualification of being True or False that is meaningful.

If no analysis of the statement allows it to meet the three abovementioned conditions, there is no Qualification of being True or False that makes sense and the proposition (or pseudo-proposition) is Not Qualifiable as being True or False.

6 - A PROPOSITION NOT QUALIFIABLE AS TRUE OR FALSE

6.1 – Let us assume that the situation below exists in correspondence with a certain physical reality:

6.1.1 – There are two chests numbered 1 and 2.
6.1.2 – One of the chests contains booty.
6.1.3 – Each chest has a sign with a statement whose truthfulness we do not assure.
6.1.4 – Nº1 says "The booty is in this chest"
Nº2 says "Only one sign is true"

6.2 – Given that both sentences seem qualifiable as T or F, let us assume they are. In that case we will have:

6.2.1 - **If** statement **Nº2 is True**, "only one is true", **then** statement **Nº1 is False**.
6.2.2 - **If** statement **Nº2 is False**, Nº1 cannot be True, because in that case we would have only one that is true and Nº2 would be True, which would be the contrary of what is assumed. **Then** statement **Nº1 is False**.
6.2.3 – As in both cases Nº1 is False ("The booty is in this chest") the booty is in chest Nº 2.

6.3 – We can point out that this solution to the problem of the booty's location is unequivocally incorrect because of the following external consideration with respect the precedent analysis:

Somebody can take the situation described in **6.1** and pose the problem again after changing the booty to the other chest, which would not violate any of the items of the problem, 6.1.1 to 6.1.4. These items continue to be formally identical and as valid as they were before.

As we have only one description for these two different situations (formally identical) it is *a priori* obvious that no reasoning on the signs could identify their contents.

What is assumed in 6.2 is therefore incorrect. Accepting that statement Nº2 is "True or False" leads to a flagrant contradiction with reality.

6.4 - Let us observe, on the other hand, that from 6.2.1 to 6.2.2, if statement Nº2 is Qualifiable as True or False**,** then Nº1, whose content was not considered there, is False, regardless of what it expresses, which is absurd.

If we consider Nº2 ("Only one sign is true") as Qualifiable as True or False, and if Nº1 were true, as for example "2 + 2 = 4", then Nº2 cannot be True ("only one is true" would be False, because there would be two that are true), nor can it be False ("only one of them is true" would be True).

In summary, if we assume that Nº2 is Qualifiable of being True or False, which means that it is either True or False, then we conclude that it is not True or False, <u>therefore it cannot be Qualifiable as being True or False.</u> (Let us assume that Nº2 is <u>Qualifiable as being True or False,</u> or else that it is not Qualifiable as being True or False. In the end, in both cases it turns to be Not Qualifiable as True or False.)

Considerations on the consistency with reality (6.3) or logical type considerations (6.4) lead us to conclude that <u>statement Nº2 is Not Qualifiable as True or False.</u>

7 - EPIMENIDES

As we feel comfortable and we like to be able to Qualify sentences as True or False when they seem to qualify as such, we will only accept the previous conclusion if we are convinced that the disqualified statement is a pseudo-statement, or violates something closer than a complex formalization.

Fortunately that happens; the disqualified statement does not even meet the minimal requirements established in 5-A and 5-B.

Moreover, the term Qualifiable as being True or False is not an attribute applicable to everything; it is applied to something we hope has some meaning. Maybe "Peanut butter is dodecaphonic" finds some meaning in the gastronomic, musical or surrealistic ambits, but the analysis using what we know as "common sense" reveals that there is no consistent meaning that can be attributed to "This statement is false", or "This proposition is undecidable".

If statement Nº1 in the previous example is True, Nº2 (“Only one is true”) means "This statement is False", which takes us directly to the liars’ paradox.

Let us then take Epimenides’ paradox the way we find it in the example we analysed:

Nº2: Statement Nº2 is False

If we explicit this by replacing “Statement Nº2” by the complete expression of the second statement, we get:

"Statement Nº 2 is False" is False,
Which (in general) is equivalent to saying:
Statement Nº2 is not False.

Let us see what is required by 5-A and 5-B. Our subject would be statement Nº2 and the attribute would be falsehood.

With the first replacement of equivalents (or presumed equivalents, if the identification of the subject were unequivocal), either our subject has changed, or what we attribute to the subject has changed. If we continue to replace “Statement Nº 2” for its complete expression, what seemed to be clearly identified will alternatively jump to the opposite places. This phrase that replaces "is False" by "is not False" does not meet what had been previously established.

Treating Epimenides’s statement as True or False (i.e. Qualifiable as True or False) leads us to the paradox of concluding that it is false if we assume it is true, and the other way round.

This contradiction is similar to the one that would result from treating Epimenides’s statement as if it were positive or negative (qualifiable >0 or <0, an assumption that would violate 5-C) and we could conclude that it is negative if we assumed it positive and vice-versa. This would not be an actual paradox, but rather a mistaken application of weak-defined or undefined qualifications.

The paradox is produced by mistakenly attributing the value of True or False to a statement that is not Qualifiable as True or False, as we saw in item **6**.

Regardless of the fact that it has no meaning, what Epimenides says is not Qualifiable as True or False; it is not True and it is not False.

8 – GÖDEL

In 1931, Kurt Gödel demonstrated that all formal systems with a certain arithmetical content are incomplete (in those systems it is possible to construct a statement in such a manner that neither the statement nor its negation are deductible within the system).

He then effectively constructed a statement with those characteristics, and using meta-mathematical considerations external to the system, he concluded (outside the formal system) that that statement was true.

The incompleteness of the formal systems was hence demonstrated, since they could contain a (true) proposition that could not be qualified as true or false within the system.

But the statement Gödel claims to be true (outside the formal system) is not true at all. Nor is it false; it is not qualifiable as true or false either within the formal system (as proved by Gödel) nor in "its natural interpretation".

The result that Gödel himself qualifies as surprising, is no longer surprising.

Even when the theorem may not be syntactically objectable, its meaning appears as follows: "If within a formal system is possible to build such an incoherent statement that cannot be *a priori* qualified as true or false, neither we will not be able to decide whether it is true or false within the limits of this system".

The claim that the statement is true (which conveys a special meaning to the fact that the formal system cannot decide whether it is true or false) occurs in Gödel's informal presentation prior to demonstration and outside any formal system.

Says Gödel,

> ***"Thus, we face a statement that claims it is undeductible. Contrary to what might seem, an enunciate of that kind is not circular at all, since it limits itself to stating that a certain formula (i.e., that obtained through a certain replacement from the formula q-ava, according to the lexicographic order) is not deductible. Only subsequently (and quite by chance) does it turn out that this formula is precisely the one expressed by that same enunciate".***
> Gödel [1; 58]

It seems obvious that the sentences:

[N° 1] Statement N° *n* is undeductible
[N° 2] Statement N° *n* is false

do not have a uniform status in all *n* values; and regardless of whether it was just by chance that the value that makes them circular was determined, it requires a specific treatment. Among other things, it requires determining whether it has any meaning. (In the second case we have seen that it does not).

Gödel expresses the delicate issue being challenged as follows:

> ***"Claiming that it is not deductible, the statement [R (q); q], immediately leads to saying that [R (q); q] is true, since [R (q); q] is not deductible (for it is not decidable). Hence, the undecidable statement in the PM system has finally been decided through meta-mathematical considerations."*** Gödel [1; 59]

Among other reasons, we say it is a delicate issue because it seems evident that a statement that claims to be undeductible, if it is indeed undeductible, then it is true. It was evident to Gödel and it seems evident to us. Even seeing that it does not meet the above-expressed requirements to be Qualifiable as True or False, it seems evident that it is true.

Reason is fortunately consistent, and as it has occurred with so many evident things that were proven wrong and replaced by something more deeply harmonious, the above does not imply that the statement is true. It is to be expected that a proposition that is not Qualifiable as True or False is not True, but as people are not used to establishing whether this qualification is appropriate (and it is hard for us to convince ourselves that this suffices), let us see it some other ways.

8.1 - The thesis that seemed obvious is:

"This Proposition is Undeductible" (**This P is U**).
If it is Undeductible, then it is True.

Let us try to explicit it by successively replacing "This P…" by the complete expression of the proposition and let us number them to identify what we are talking about:

K0	This P			
K1	This P is U	K0 is U		
K2	"This P is U" is U	K1 is U	"K0 is U " is U	
K3	"(This P is U) is U" is U	K2 is U	"K1 is U" is U	"(K0..

8.1.1 - The thesis in 8.1, and what Gödel says is literally:
If K1 is U, then K1 is True

8.1.2 - K1 is U is stated by K2, so the above equals:
If K2 is T, then K1 is T

8.1.3 – Not only does nothing allow us to state if an expression is True, then the previous expression is also True, but (regardless the fact they may have some meaning, or that they are Not Qualifiable as T or F) each one of them is contradictory with the one next to it, since any of them would be deductible if we proved that the previous one is undeductible, and in that case the next one would be False.

Then if ("This P is U") is undeductible, the truth is
only ("This P is U" is undeductible),
and not "This P is U".

It is a mistake to believe that because one is True, so is the other one because it is the same. This mistake is shown by the fact that, if one is deductible, what can be stated about the other, is precisely the contrary.

8.2 – Let us see it another way, by comparing the sentence we are discussing with sentences that seem to have similar structure.
Let us consider the three sentences below:

1. This phrase has five words
2. This was printed with black ink
3. This proposition is undeductible

If we accept that the first sentence has five words, the second one was printed in black ink, and the third one is undeductible, then we draw the apparently obvious and evident conclusion that the three are true.

This simple syllogism is the one applied by Gödel in the paragraph mentioned above [1;59] outside the formal system.

We will see that in the third case the conclusion has been drawn erroneously.

For the purpose of identification, let us name the terms distinguishing their position in the syllogism:

Hypothesis 1 - A1 says that A2 has property P
Hypothesis 2 - A3 has property Q
Thesis - A4 is true

When Q implies P, A3 = A2 and A4 = A1, the conclusion is valid.
The syllogism then says:
H 1 - A1 says that A2 has property P
H 2 - A2 has property P
T - A1 is true

For the first statement, if we identify sentences as:
A2 = This phrase has five words

H1	A1 = A2 has five words
H2	A2 has five words
T	A1 is True
T'	(A2 has five words) is True
T"	("This phrase has five words" has five words) is True
This syllogism does not state that A2 is True	

<u>**If**</u> we can add this equivalence:
This phrase = "This phrase has five words ",
and we replace is T", we get:
(This phrase has five words) is True
<u>**then**</u> **A2 is True**.

For the third statement, identifying sentences as:
G2 = This proposition is Undeductible

H1	G1 = G2 is Undeductible
H2	G2 is Undeductible
T	G1 is True
T'	(G2 is Undeductible) is True
T''	("This proposition is Undeductible" is Undeductible) is True
	This syllogism does not state that G2 is True

We cannot add similar equivalences to the previous case because:

This Proposition	This Proposition is Undeductible	≠
If this one is deductible and hence True,	it implies this other one is False	

"This Proposition is Undeductible" is Undeductible	This Proposition is Undeductible	≠
Gödel demonstrates this, then	this other one is non demonstrable	

Then, we cannot conclude that G2 is True.

8.3 – The syllogism expressed by Gödel [1;59] is literally the following with
G = [R(q);q]

H1 G says that G is Undeductible
H2 G is Undeductible
T G is True

The conclusion **[T]**, refers to one of the two "G"s from hypotesis **[H1]**, which are different. They are different because as Gödel would say, [1;173] the first G is in a meta-language in reference to the second one; and what turns out decisive, is that they are different because both are contradictory. This is shown by the fact that, if the second one is deductible, -and hence, True-, then the first one is False.

The conclusion **[T]** refers to the first G of **[H1]** and not to the second. The positive statement **[H2]** "G is Undeductible", is True, but is not True the "G" proposition included in **[H2]**.

Identifying these two different "G"s as G1 and G2, we are back in the the previous analyzed case.

8.4 – The various analyses then show, that the apparently obvious syllogism used by Gödel to conclude that the quoted proposition [1; 59] is true, is not correct. The quoted proposition is not true (nor is it false).

By concealing the precise subject we refer to, recurrence led us to think that a certain statement was true when it was actually not true at all.

As the statement proposing it is undeductible is not true, the strange surprise originated by the VI Theorem of Incompleteness disappears.

Regardless of the formal aspect of the demonstration, demonstrating incompleteness in a system that cannot decide the validity of a true proposition is conceptually very different from being able to construct a statement in this system that does not make sense and is neither true nor false. Consequently, *a priori* neither that statement nor its denial can be deduced in the system.

Since it becomes clear that something that looks like a statement either is not such, or at least it is not qualifiable as true or false, in a useful system we cannot expect but that undecidablity, and the demonstration of that undecidability is a consequence of this system's consistency and adequacy.

9 - CONCLUSIONS

9.1 – There are sentences and/or pseudo-sentences in natural language that are not universally qualifiable as True or False. If that character is not evident, we must make use of adequate tools to disclose it.

9.2 – The liar's paradox appears if we make the mistake of treating Epimenides's statement as being qualifiable as True or False, when it is actually not qualifiable. In the terms of the example used in (6.3), if the proposition were "True or False" the booty would always be in chest Nº 2, in flagrant contradiction with reality.

9.3 – Prior to his theorem on incompleteness, Gödel claims that an undecidable statement is true, which conveys a special meaning to the fact of being undecidable. The analysis shows that such statement is not True.

REFERENCES

[1] - Kurt Gödel, Obras completas
Alianza Universidad 286. Alianza Editorial S.A.
Madrid, 1981

[2] - Rudolf Carnap, The logical syntax of language
Routledge & Kegan Paul Ltd.
London, 1937

[3] - Rudolf Carnap , Ciencia y metafísica ante el análisis lógico del lenguaje
Ediciones de La Casa del Estudiante
Montevideo, 1976

[4] - Paul Watzlawick, Cambio
Biblioteca de Psicología 35. Editorial Herder
Barcelona, 1985

[5] - R. Montague, English as a formal language
Yale University Press
Yale, 1974

[6] - D. Davidson, Truth and meaning
Philosophical Logic
Dordrecht, 1959

[7] - Helios Pazos, Determinismo y causalidad
Pettirossi Hnos.
Montevideo, 1983

References [work; page]